

MoC - online
1a prova in itinere
23/11/2012

COGNOME _____
NOME _____
MATRICOLA _____
FIRMA _____

Disuguaglianze

1) $\sqrt{9x^2 - 16} \geq x + 3$

2) $\sqrt[2]{x-1} < \sqrt[3]{x^2-1}$

3) $\frac{2^{2x} - 2^x - 12}{3^{-2x} - 4 \cdot 3^{-x} + 3} \geq 0$

4) $0 < \arctan \frac{x-1}{x+1} \leq \frac{\pi}{3}$

5) $\frac{|x|}{x-1} < 1 + \frac{2}{x-1}$

6) $\frac{x^4 - 2x - 1}{x^2 - 2x - 1} < 0$

Complessi

1) Scrivere in forma trigonometrica

$$z = \left(\frac{2-2i}{1+i} \right)^4$$

2) Scrivere in forma algebrica

$$z = \left(\frac{2-\sqrt{3}i}{2+\sqrt{3}i} \right)^2 - \left(\frac{2+\sqrt{3}i}{2-\sqrt{3}i} \right)^2$$

3) Dato un numero z con $|z|=e$, $\arg z = \frac{\pi}{4}$
calcolare e disegnare $\sqrt[4]{z}$ e $\log z$

MdC - 1^a prova in itinere - ore 23/11/2012

① $\sqrt{9x^2 - 16} \geq x + 3$ CE: $9x^2 - 16 \geq 0$
 $(-\infty, -4/3] \cup [4/3, +\infty)$

a) se $x + 3 < 0$ cioè $x < -3$ $(1^a \text{ me} > 0) \geq (2^a \text{ me} < 0)$
OK

b) se $x \geq -3$ $1^a \text{ me} \geq 0$ e $2^a \geq 0 \Rightarrow$ posso quadrare

$$9x^2 - 16 \geq x^2 + 6x + 9$$

$$8x^2 - 6x - 25 \geq 0$$

$$x \leq \alpha \text{ e } x \geq \beta$$

$$x = \frac{3 \pm \sqrt{9 + 200}}{8}$$

$$\begin{cases} \sim 3(\beta) \\ \sim -3/2(\alpha) \end{cases}$$

a) $\begin{array}{c} -3 \\ \text{soluz?} \end{array}$ $\frac{3 + \sqrt{209}}{8}$

b) $\begin{array}{c} \text{soluz?} \end{array}$

$$\frac{3 + \sqrt{209}}{8}$$

soluz?

CE $\begin{array}{c} -4/3 \\ \text{OK soluz} \end{array}$

$\frac{4}{3}$ $\begin{array}{c} \text{OK soluz} \end{array}$

$$x \leq \frac{3 - \sqrt{209}}{8}$$

$$x \geq \frac{3 + \sqrt{209}}{8}$$

②

$$\sqrt{x-1} < \sqrt[3]{x^2-1}$$

CE $x \geq 1$

se $x \geq 1$ (ovvero nel CE) $(1^a \text{ me} \geq 0) < (2^a \text{ me} \geq 0)$
 posso elevare alla 6^a

$$(x-1)^3 < (x-1)^2(x+1)^2$$

$x=1$ non è soluzione e $(x-1)^2 \geq 0 \Rightarrow$

$$x-1 < (x+1)^2$$

$$x^2+x+2 > 0$$

$$\Delta = 1-8 < 0$$

\Rightarrow il trinomio è positivo

Per cui la soluzione coincide con il CE $= \{1\}$

$$\textcircled{3} \quad \frac{2^{2x} - 2^x - 12}{3^{-2x} - 4 \cdot 3^{-x} + 3} \geq 0$$

CE: denominatore $\neq 0$

$$2^x = \frac{1 \pm \sqrt{1+48}}{2}$$

$$= \frac{1 \pm 7}{2} = \begin{cases} 4 \\ -3 \end{cases}$$

$$2^x = 4$$

$$x = 2$$

$$2^x = -3$$

$$2^x > 0 \quad \forall x$$

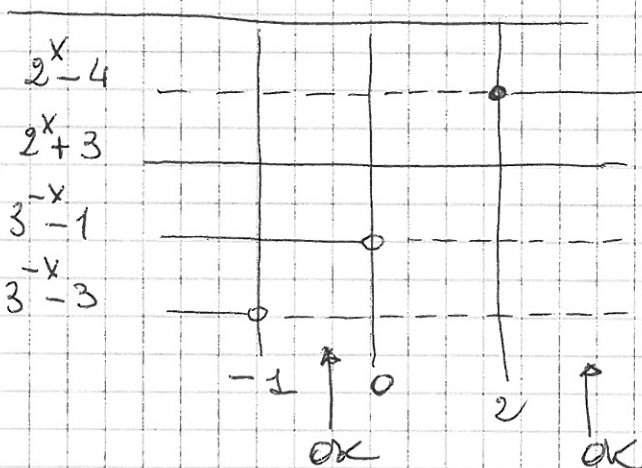
$$\frac{(2^x - 4)(2^x + 3)}{(3^{-x} - 1)(3^{-x} - 3)} \geq 0$$

$$3^{-x} = 2 \pm \sqrt{4-3} < 3$$

$$3^{-x} \neq 3 \quad \text{e} \quad 3^{-x} \neq 1$$

$$x \neq -1$$

$$x \neq 0$$



$$-1 < x < 0 \quad \text{e} \quad x \geq 2$$

$$\textcircled{4} \quad 0 < \operatorname{arctg} \frac{x-1}{x+1} \leq \frac{\pi}{3}$$

$$\text{CE} \quad x \neq -1$$

$$0 < \frac{x-1}{x+1} \leq \operatorname{tg} \frac{\pi}{3} = \sqrt{3}$$

$$\left\{ \begin{array}{l} \frac{x-1}{x+1} > 0 \\ \frac{x-1}{x+1} \leq \sqrt{3} \end{array} \right.$$

$$x < -1 \quad x > 1$$

$$x \leq -\frac{\sqrt{3}+1}{\sqrt{3}-1} \quad x > -1$$

\Rightarrow

$$\frac{x-1}{x+1} \leq \frac{\sqrt{3}x + \sqrt{3}}{x+1}$$

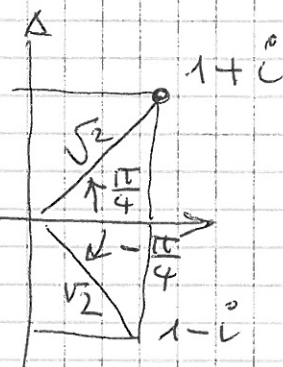
$$\frac{x(\sqrt{3}-1) + (\sqrt{3}+1)}{x+1} > 0$$

$$x \leq -\frac{\sqrt{3}+1}{\sqrt{3}-1} \quad \text{e} \quad x > 1$$

Complessi

1) $\left(\frac{2-2i}{1+i}\right)^4 = 16 \left(\frac{1-i}{1+i}\right)^4 =$

$$16 \left(\frac{\sqrt{2} \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right)}{\sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)} \right)^4$$



$$= 16 \left(\cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right) \right)^4$$

$$= 16 \left(\cos(-2\pi) + i \sin(-2\pi) \right) = 16$$

2)

$$z = \left(\frac{2-\sqrt{3}i}{2+\sqrt{3}i} \right)^2 - \left(\frac{2+\sqrt{3}i}{2-\sqrt{3}i} \right)^2 =$$

$$= \left(\frac{2-\sqrt{3}i}{2+\sqrt{3}i} + \frac{2+\sqrt{3}i}{2-\sqrt{3}i} \right) \left(\frac{2+\sqrt{3}i}{2+\sqrt{3}i} - \frac{2+\sqrt{3}i}{2-\sqrt{3}i} \right) =$$

$$= \frac{(2-\sqrt{3}i)^2 + (2+\sqrt{3}i)^2}{4+3} \cdot \frac{(2-\sqrt{3}i)^2 - (2+\sqrt{3}i)^2}{4+3} =$$

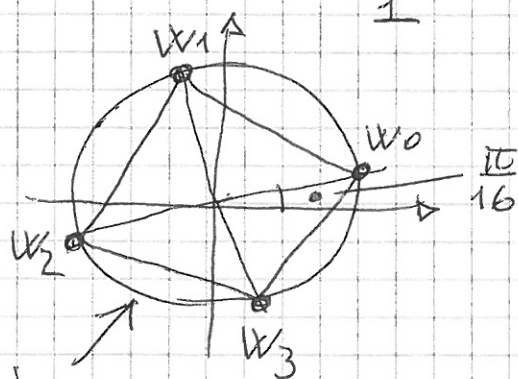
$$= \frac{4-3-4\sqrt{3}i+4-3+4\sqrt{3}i}{7} \cdot \frac{4-3-4\sqrt{3}i-4+3-4\sqrt{3}i}{7} =$$

$$= \frac{-16\sqrt{3}i}{49} = 0 + i \frac{16\sqrt{3}}{49}$$

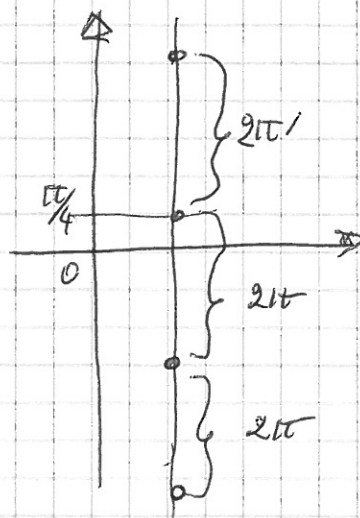
3) $|z| = e \quad \arg z = \frac{\pi}{4}$

$$\sqrt[4]{z} = e^{1/4} \left(\cos \frac{\frac{\pi}{4} + 2k\pi}{4} + i \sin \frac{\frac{\pi}{4} + 2k\pi}{4} \right) \quad k=0,1,2,3$$

$$\log z = \underbrace{\log e}_1 + i \left(\frac{\pi}{4} + 2k\pi \right) \quad k=0, \pm 1, \dots$$



raggio $\sqrt[4]{e}$



⑤

$$\frac{|x|}{x-1} < 1 + \frac{2}{x-1}$$

$$CE \quad x \neq 1$$

a) se $x \geq 0 \quad |x| = x$

$$\frac{x}{x-1} < \frac{x-1+2}{x-1}$$

$$\frac{2}{x-1} > 0 \quad \boxed{x > 1}$$

b) se $x < 0 \quad |x| = -x$

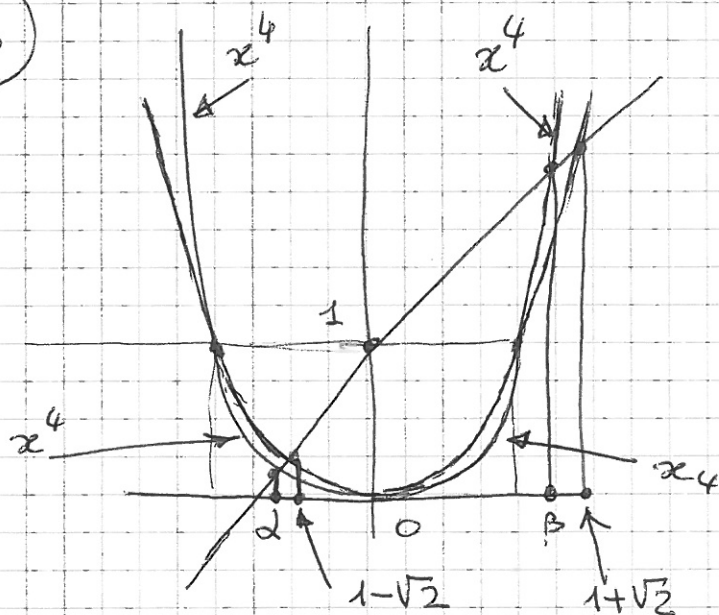
$$\frac{-x}{1-x} < \frac{x-1+2}{x-1}$$

$$\frac{2x-1}{x-1} > 0$$

$$x < 1/2 \quad x > 1 \Rightarrow \boxed{x < 0}$$

$$(a+b) \Rightarrow x < 0, x > 1$$

⑥



$$\frac{x^4 - (2x+1)}{x^2 - (2x+1)} < 0$$

$$\begin{cases} y = x^4 \\ y = 2x+1 \end{cases} \Rightarrow \begin{matrix} x = \alpha \\ x = \beta \end{matrix}$$

$$x^2 - 2x - 1 = 0$$

$$x = 1 \pm \sqrt{2}$$

$$Num > 0$$

$$Den > 0$$

