Recupero II Compitino
MdC / Fond

16/3/2013

1) $\sum \dfrac{(n!)^3 n^6}{(3n)!}$

2) $\sum \left(\dfrac{1}{\sqrt{n+1}-1} - \dfrac{1}{\sqrt{n+1}+1} \right)$

3) $\sum (-1)^n \cdot \sqrt{n} \cdot 2^{kn}$ con k reale

4) $\lim\limits_{n \to +\infty} \left(\sqrt[3]{27n^3 - 9n} - 3n \right)$

5) $\lim\limits_{n \to +\infty} \left(1 - \dfrac{1}{n} \right)^{\frac{n^2}{tg(1/n)}}$

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1) $\sum \frac{(n!)^3 n^6}{(3n)!}$

$$\frac{a_{n+1}}{a_n} = \frac{\overbrace{\cancel{((n+1)!)^3}}^{(n+1)^3} (n+1)^6}{\underbrace{\cancel{(3(n+1))!}}_{(3n+1)(3n+2)(3n+3)}} \cdot \frac{\cancel{(3n)!}}{\cancel{(n!)^3} \cdot n^6} =$$

$$= \frac{(n+1)^3}{(3n+1)(3n+2)(3n+3)} \cdot \underbrace{\left(1+\frac{1}{n}\right)^6}_{\downarrow 1} \longrightarrow \frac{1}{27} < 1$$

$\Longrightarrow \sum$ converge

2) $\sum \left(\frac{1}{\sqrt{n+1}-1} - \frac{1}{\sqrt{n+1}+1} \right)$

$$a_n = \frac{\cancel{\sqrt{n+1}} + 1 - \cancel{\sqrt{n+1}} + 1}{(n \cancel{+1}) \cancel{-1}} = \frac{2}{n}$$

si comporta come la serie armonica

$\Longrightarrow$ diverge

3) $\sum (-1)^n \sqrt{n}\, 2^{kn} \qquad k \in \mathbb{R}$

condiz necessaria per la convergenza
$a_n \to 0$, questo implica $k<0$

$\boxed{k<0}$
è a segni alterni $\sum (-1)^n a_n$
$a_n \to 0$
è vero che $a_n > a_{n+1}$? cioè $\{a_n\}$ è monotona?

$$\sqrt{n}\cdot 2^{kn} - \sqrt{n+1}\cdot 2^{k(n+1)} =$$

$$= 2^{kn}\left(\sqrt{n} - 2^k\sqrt{n+1}\right)$$

ma $\sqrt{n} > 2^k \sqrt{n+1}$

infatti

$$n > 2^{2k} n + 2^{2k}$$

è un valore $0< \quad <1$

$$n\left(1-2^{2k}\right) > 2^{2k}$$

$$n > \frac{2^{2k}}{1-2^{2k}} \quad \text{definitivamente}$$

quindi a_n è definitivamente monotona

$\Longrightarrow$ per k negativo la serie
converge per il criterio
di Leibnitz

4) $\lim\limits_{n\to+\infty}\left(\sqrt[3]{27n^3-9n}-3n\right)$

$$\sqrt[3]{27n^3-9n}-3n = 3n\left(\underbrace{\sqrt[3]{1-\frac{1}{3n^2}}-1}\right)=$$

$$\Rightarrow \sim \cancel{3n}\cdot\frac{1}{\cancel{3}}\cdot\left(-\frac{1}{3n^2}\right)= \qquad \sim \frac{1}{3}\cdot\left(-\frac{1}{3n^2}\right)$$

$$= -\frac{1}{3n} \to 0^-$$

5) $\lim\limits_{n\to+\infty}\left(1-\frac{1}{n}\right)^{\frac{n^2}{tg\, 1/n}}$

$$\left(1-\frac{1}{n}\right)^{\frac{n^2}{tg\, 1/n}} = \underbrace{\underbrace{\left(1-\frac{1}{n}\right)^{-n}}_{\downarrow\ e}{}^{\underbrace{\frac{-n}{tg\, 1/n}}_{\downarrow\ -\frac{\infty}{0}}}}_{e^{-\infty}}$$

$$\underbrace{e^{-\infty}}_{0^+}$$