MdC/Foud Recupero 1° compitino

16/3/2013

1) $x^4 - 2\left(x^2 + x \cdot |x|\right) + \frac{x}{|x|} \geq 0$

2) $\sqrt{\sqrt{x^2+3x} - x - 1} \geq 1$

3) $\log \frac{x^2-4}{x} > 1$

4) Calcolare e disegnare $\sqrt[3]{-27}$ in $\mathbb{C}$

5) Senza (mai!) passare in forma algebrica calcolare

$$\left(\frac{1-i}{\sqrt{3}+i}\right)^5$$

Recupero I° compitino 16/3/2013

1) $x^4 - 2(x^2 + x \cdot |x|) + \frac{x}{|x|} \geq 0$

C.E. $x \neq 0$

se $x > 0$ $\quad |x| = x$

$\Rightarrow \quad x^4 - 4x^2 + 1 \geq 0$

$x^2 = 2 \pm \sqrt{4-1} = 2 \pm \sqrt{3}$

$x^2 \leq 2 - \sqrt{3} \quad x^2 \geq 2 + \sqrt{3}$

$$\begin{cases} -\sqrt{2-\sqrt{3}} \leq x \leq \sqrt{2-\sqrt{3}} \;;\; x \leq -\sqrt{2+\sqrt{3}} \;;\; x \geq \sqrt{2+\sqrt{3}} \\ x > 0 \end{cases}$$

$0 < x \leq \sqrt{2-\sqrt{3}} \;;\; x \geq \sqrt{2+\sqrt{3}}$ (A)

se $x < 0$ $\quad |x| = -x$

$\Rightarrow \quad x^4 - 1 \geq 0$

~~$x^2 \leq -1$~~ ; $x^2 \geq 1$

$$\begin{cases} x \leq -1 \quad x \geq 1 \\ x < 0 \end{cases}$$

$x \leq -1$ (B)

Soluz: A e B

2) $\sqrt{\sqrt{x^2+3x}-x-1} \geq 1$

C.E.: $x^2+3x \geq 0$ e $\sqrt{x^2+3x}-x-1 \geq 0$

prima di fare i conti osserviamo che I°m ≥ 0 nel C.E. e II°m ≥ 0 quindi per risolvere eleveremo al quadrato entrambi i membri

$\sqrt{x^2+3x}-x-1 \geq 1$ ‖ questo rende inutile

$$\begin{cases} x^2+3x \geq 0 \\ \sqrt{x^2+3x} \geq x+2 \end{cases} \Rightarrow x \leq -3 \; ; \; x \geq 0$$

se $x+2<0$ (cioè $x<-2$) I°m ≥ 0, II°m <0 $\Rightarrow$ ok

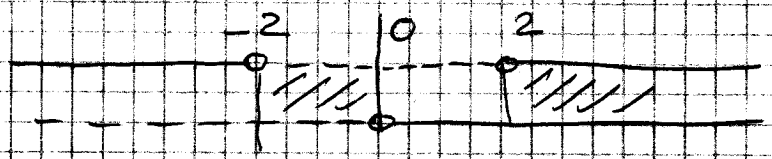
se $x+2 \geq 0$ (cioè $x \geq -2$) $x^2+3x \geq x^2+4x+4$

$x \leq -4$ non rientra nella condizione

$$\begin{cases} x \leq -3 \; ; \; x \geq 0 \\ x<-2 \end{cases} \Rightarrow \boxed{x \leq -3}$$

3) $\log \frac{x^2-4}{x} > 1$

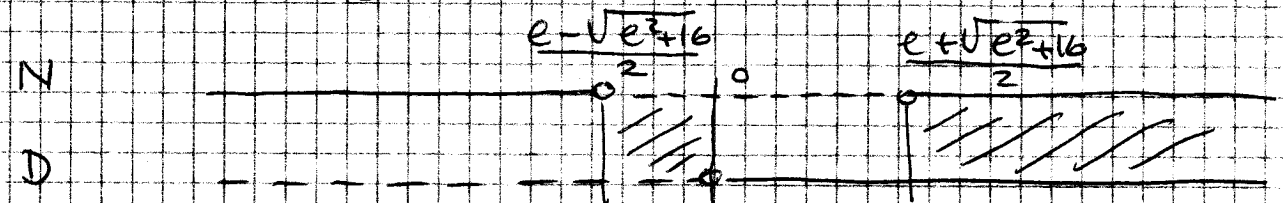
CE: $\frac{x^2-4}{x} > 0$ e $x \neq 0$



$-2 < x < 0$ e $x > 2$

$\frac{x^2-4}{x} > e \qquad \frac{x^2-ex-4}{x} > 0$

$x = \frac{e \pm \sqrt{e^2+16}}{2}$



CE:

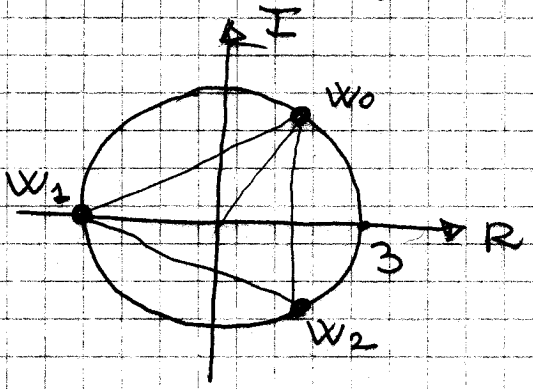
$\Rightarrow \quad \frac{e-\sqrt{e^2+16}}{2} < x < 0$ e $x > \frac{e+\sqrt{e^2+16}}{2}$

4) Calcolare e disegnare $\sqrt[3]{-27}$

$z = -27 \Rightarrow |z| = 27, \arg z = \pi$

$$\Rightarrow \sqrt[3]{z} = 3\left(\cos\frac{\pi+2k\pi}{3} + i\,\text{sen}\,\frac{\pi+2k\pi}{3}\right)$$

$k = 0, 1, 2$



5) Senza (mai!) passare in forma algebrica calcolare

$$\left(\frac{1-i}{\sqrt{3}+i}\right)^5$$

$|1-i| = \sqrt{2} \qquad \arg(1-i) = -\frac{\pi}{4}$

$|\sqrt{3}+i| = 2 \qquad \arg(\sqrt{3}+i) = \frac{\pi}{6}$

$$\left(\frac{\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\,\text{sen}\left(-\frac{\pi}{4}\right)\right)}{2\left(\cos\frac{\pi}{6}+i\,\text{sen}\frac{\pi}{6}\right)}\right)^5 =$$

$$= \left(\frac{\sqrt{2}}{2}\left(\cos\left(-\frac{\pi}{4}-\frac{\pi}{6}\right)+i\,\text{sen}\left(-\frac{\pi}{4}-\frac{\pi}{6}\right)\right)\right)^5 =$$

$$= \left(\frac{\sqrt{2}}{2}\right)^5\left(\cos\left(5\cdot\left(-\frac{5}{12}\pi\right)\right)+i\,\text{sen}\left(5\cdot\left(-\frac{5}{12}\pi\right)\right)\right) =$$

$$= \frac{\sqrt{2}}{8}\left(\cos\left(-\frac{\pi}{12}\right)+i\,\text{sen}\left(-\frac{\pi}{12}\right)\right)$$