26/1/2013 MdC e FMdC (2)

1) $$\lim_{n\to+\infty} \frac{\log\left(1+\frac{\sqrt{n}}{2n+1}\right)}{e^{\frac{3\sqrt{n}+1}{4n}}-1}$$

2) $$\lim_{n\to+\infty} \left(2-\sqrt{1+\frac{n}{n^2-1}}\right)^{\frac{1}{1+\sqrt{1+\frac{n}{n^2-1}}}}$$

3) $$\lim_{n\to+\infty} \left(\frac{3n}{3n-2}\right) \operatorname{sen}^2 \frac{1}{2n}$$

4) $$\lim_{n\to+\infty} \frac{\left(\cos^4 \frac{2n}{n^2+3}\right)^{1/5}-1}{n^\alpha}$$ quando esiste finito? e quanto vale?

5) $$\lim_{n\to+\infty} \frac{\operatorname{sen}^2\frac{1}{n}+n^{\frac{1}{1000}}-1-\log^{1000}(n^{1000})}{\operatorname{tg}^2\frac{n^3-1}{n^4-1}+n^{\frac{1}{500}}}$$

6) $$\sum\left(\frac{1+k}{1-k}\right)^n$$ per quali valori di k converge

7) $$\sum \frac{n^2\log^2 n+1}{n^\alpha \log^2 n+\alpha}$$ per quali valori di α converge?

26/1/2013 (2)

1) $\log(1+\varepsilon) \sim \varepsilon$ e $e^{\varepsilon}-1 \sim \varepsilon$

2) $$\left(2 - \sqrt{1+\frac{n}{n^2-1}}\right) = \left(1 + \left(1 - \sqrt{1+\frac{n}{n^2-1}}\right)\right)$$

quindi

$$\left(2 - \sqrt{1+\frac{n}{n^2-1}}\right) = \underbrace{\left(1 + \left(1 - \sqrt{1+\frac{n}{n^2-1}}\right)\right)^{\frac{1}{1-\sqrt{\ }}}}_{\downarrow e}{}^{\cdot \frac{1-\sqrt{\ }}{1+\sqrt{\ }}}$$

3) ~~[illegible]~~ $$\binom{3n}{3n-2} = \frac{(3n)!}{(3n-2)!\,\underbrace{(3n-(3n-2))!}_{2!}} =$$

$$= \frac{(3n)(3n-1)\cancel{(3n-2)!}}{\cancel{(3n-2)!}\ 2!} = \frac{9n^2-3n}{4}$$

$$\operatorname{sen}^2\left(\frac{1}{2n}\right) \sim \left(\frac{1}{2n}\right)^2 = \frac{1}{4n^2}$$

4) $$\underbrace{\left(1 - \underbrace{\left(1-\cos^2\frac{2n}{n^2+3}\right)}_{\varepsilon}\right)^{1/5} - 1}$$

$$\sim$$

$$\frac{1}{5}\left(1-\cos^2\frac{2n}{n^2+3}\right) = \frac{1}{5}\left(1+\cos^2\frac{2n}{n^2+3}\right)\left(1-\underbrace{\operatorname{sen}^2\frac{2n}{n^2+3}}_{\sim\left(\frac{2n}{n^2+3}\right)^2}\right)$$

5) $$\frac{\overset{\uparrow 0}{\operatorname{sen}^2\frac{1}{n}} + n^{\frac{1}{1000}} - 1 - \cancel{\log_{1000}}(n^{1000})}{\underbrace{\operatorname{tg}^2\left(\frac{n^3-1}{n^4-1}\right)}_{\sim \frac{1}{n} \downarrow 0} + n^{\frac{1}{500}}}$$

6) converge per $\left|\frac{1+k}{1-k}\right| < 1$

cioè soluz sistema

$$\begin{cases} \frac{1+k}{1-k} > -1 \\ \frac{1+k}{1-k} < 1 \end{cases}$$

7) $\sum a_n$ con $a_n \sim \frac{n^2 \cancel{\log^2 n}}{n^\alpha \cancel{\log^2 n}} = \frac{1}{n^{\alpha-2}}$

$\uparrow$

serie armonica generalizzata

$$\sum \frac{1}{n^k} \begin{cases} \text{conv per } k>1 \\ \text{div. per } k \leq 1 \end{cases}$$