MdC – 1° prova in itinere – on line 23/11/2012

(1) $\sqrt{9x^2-16} \geq x+3$ CE : $9x^2-16 \geq 0$

$(-\infty, -4/3] [4/3, +\infty)$

a) se $x+3<0$ cioè $x<-3$ $(1^\circ m > 0) \geq (2^\circ m < 0)$

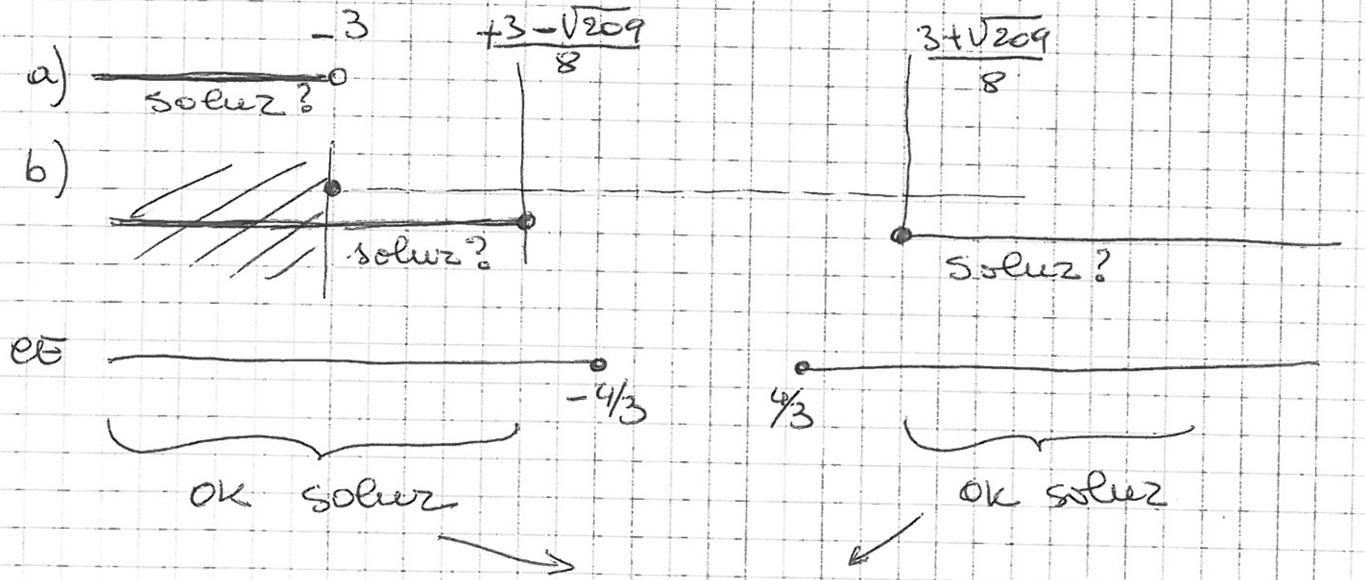
OK

b) se $x \geq -3$ $1^\circ m \geq 0$ e $2^\circ \geq 0$ $\Rightarrow$ posso quadrare

$9x^2-16 \geq x^2+6x+9$

$8x^2-6x-25 \geq 0$ $x = \dfrac{3 \pm \sqrt{9+200}}{8}$ $\sim 3 (\beta)$, $\sim -3/2 (\alpha)$

$x \leq \alpha$ e $x \geq \beta$



$x \leq \dfrac{3-\sqrt{209}}{8}$ $x \geq \dfrac{3+\sqrt{209}}{8}$

(2) $\sqrt{x-1} < \sqrt[3]{x^2-1}$ CE $x \geq 1$

se $x \geq 1$ (ovvero nel CE) $(1^\circ m \geq 0) < (2^\circ m \geq 0)$

posso elevare alla 6ª

$(x-1)^3 < (x-1)^2 (x+1)^2$

$x=1$ non è soluzione e $(x-1)^2 \geq 0$ $\Rightarrow$

$x-1<(x+1)^2 \qquad x^2+x+2>0 \qquad \Delta=1-8<0$

$\Rightarrow$ il trinomio è positivo

quindi la soluz coincide con il CE $-\{1\}$

(3) $$\frac{2^{2x}-2^{x}-12}{3^{-2x}-4\cdot 3^{-x}+3}\geq 0$$

CE: denominatore $\neq 0$

$$3^{-x}=2\pm\sqrt{4-3} \Big\langle \begin{matrix} 3 \\ 1 \end{matrix}$$

$3^{-x}\neq 3$ e $3^{-x}\neq 1$

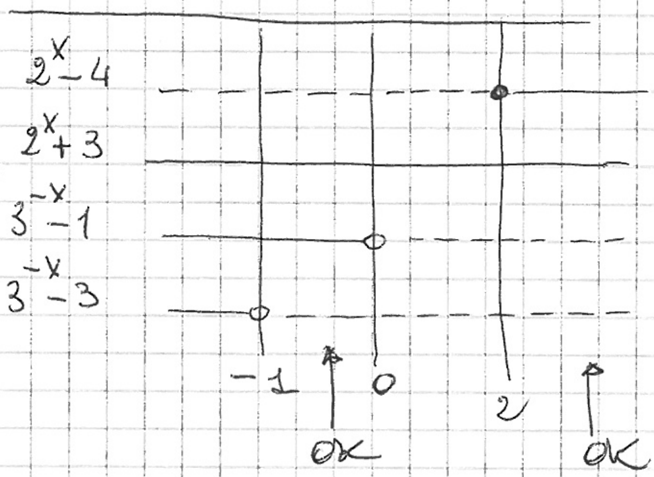
$x\neq -1 \qquad x\neq 0$

$$2^x=\frac{1\pm\sqrt{1+48}}{2}=\frac{1\pm 7}{2}=\Big\langle \begin{matrix} 4 \\ -3 \end{matrix}$$

$2^x=4 \qquad 2^x=-3$

$x=2 \qquad 2^x>0 \ \forall x$

$$\frac{(2^x-4)(2^x+3)}{(3^{-x}-1)(3^{-x}-3)}\geq 0$$



$-1<x<0$ e $x\geq 2$

(4) $$0<\operatorname{artg}\frac{x-1}{x+1}\leq\frac{\pi}{3}$$

CE $x\neq -1$

$$0<\frac{x-1}{x+1}\leq \operatorname{tg}\frac{\pi}{3}=\sqrt{3}$$

$$\left\{\begin{array}{lll} \dfrac{x-1}{x+1}>0 & x<-1 & x>1 \\ \dfrac{x-1}{x+1}\leq\sqrt{3} & x\leq-\dfrac{\sqrt{3}+1}{\sqrt{3}-1} & x>-1 \end{array}\right\}\Rightarrow$$

$$\frac{x-1}{x+1}\leq\frac{\sqrt{3}x+\sqrt{3}}{x+1}$$

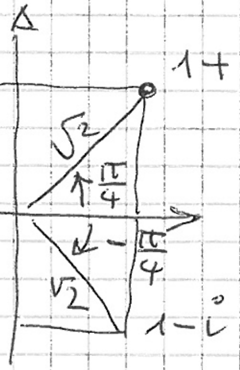
$$\frac{x(\sqrt{3}-1)+(\sqrt{3}+1)}{x+1}>0$$

$x\leq-\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ e $x>1$

Complessi

(1)

$$\left(\frac{2-2i}{1+i}\right)^4 = 16\left(\frac{1-i}{1+i}\right)^4 =$$



$$16\left(\frac{\cancel{\sqrt{2}}\left(\cos\left(-\frac{\pi}{4}\right)+i\,\mathrm{sen}\left(-\frac{\pi}{4}\right)\right)}{\cancel{\sqrt{2}}\left(\cos\left(\frac{\pi}{4}\right)+i\,\mathrm{sen}\left(\frac{\pi}{4}\right)\right)}\right)^4$$

$$= 16\left(\cos\left(-\frac{\pi}{2}\right)+i\,\mathrm{sen}\left(-\frac{\pi}{2}\right)\right)^4$$

$$= 16\left(\cos(-2\pi)+i\,\cancel{\mathrm{sen}(-2\pi)}\right) = 16$$

(2)

$$z=\left(\frac{2-\sqrt{3}i}{2+\sqrt{3}i}\right)^2-\left(\frac{2+\sqrt{3}i}{2-\sqrt{3}i}\right)^2=$$

$$=\left(\frac{2-\sqrt{3}i}{2+\sqrt{3}i}+\frac{2+\sqrt{3}i}{2-\sqrt{3}i}\right)\left(\frac{2+\sqrt{3}i}{2+\sqrt{3}i}-\frac{2+\sqrt{3}i}{2-\sqrt{3}i}\right)=$$

$$=\frac{(2-\sqrt{3}i)^2+(2+\sqrt{3}i)^2}{4+3}\cdot\frac{(2-\sqrt{3}i)^2-(2+\sqrt{3}i)^2}{4+3}=$$

$$=\frac{4-3-\cancel{4\sqrt{3}i}+4-3+\cancel{4\sqrt{3}i}}{7}\cdot\frac{\cancel{4-3}-4\sqrt{3}i-\cancel{4+3}-4\sqrt{3}i}{7}$$

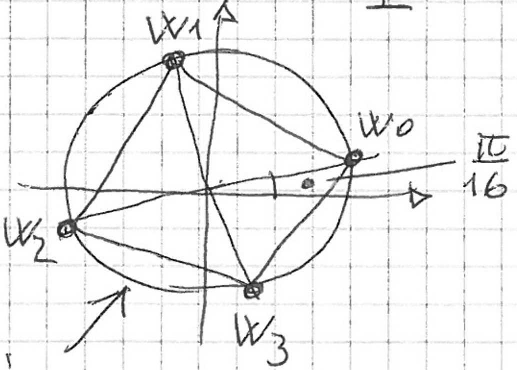
$$=\frac{-16\sqrt{3}i}{49}=0-i\,\frac{16\sqrt{3}}{49}$$

(3)

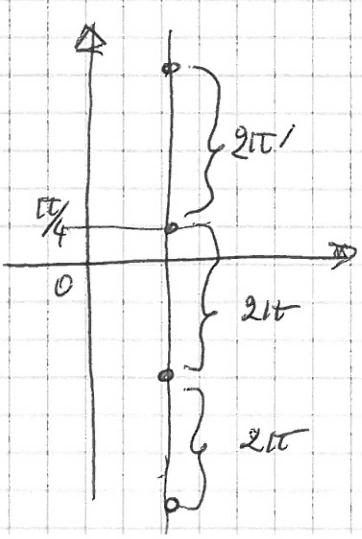
$$|z|=e \qquad \arg z=\frac{\pi}{4}$$

$$\sqrt[4]{z}=e^{1/4}\left(\cos\frac{\frac{\pi}{4}+2k\pi}{4}+i\,\mathrm{sen}\frac{\frac{\pi}{4}+2k\pi}{4}\right)\qquad k=0,1,2,3$$

$$\log z=\underbrace{\log e}_{1}+i\left(\frac{\pi}{4}+2k\pi\right)\qquad k=0,\pm1,\ldots$$



raggio $\sqrt[4]{e}$



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$$\frac{|x|}{x-1} < 1 + \frac{2}{x-1} \qquad \text{CE } x \neq 1$$

a) se $x \geq 0$ $\quad |x| = x$

$$\frac{\cancel{x}}{x-1} < \frac{\cancel{x}-1+2}{x-1} \qquad \frac{2}{x-1} > 0 \qquad \boxed{x>1}$$

b) se $x<0$ $\quad |x| = -x$

$$\frac{x}{1-x} < \frac{x-1+2}{x-1} \qquad \frac{2x-1}{x-1} > 0$$

$$x < 1/2 \quad x > 1 \implies \boxed{x<0}$$

$$(a+b) \implies x<0 \,,\; x>1$$

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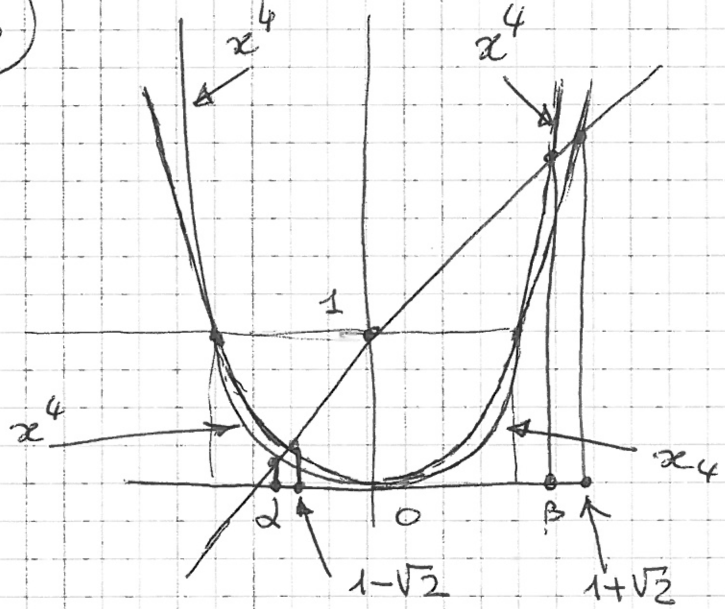

$$\frac{x^4-(2x+1)}{x^2-(2x+1)} < 0$$

$$\begin{cases} y = x^4 \\ y = 2x+1 \end{cases} \implies \begin{array}{l} x=\alpha \\ x=\beta \end{array}$$

$$x^2-2x-1=0$$

$$x = 1 \pm \sqrt{2}$$

Num > 0

Den > 0