II comp. 10|1|2014

1)

$$\lim_{n\to+\infty} \binom{2n+1}{2n-1} \operatorname{tg} \frac{1}{3n^2+n} = \frac{2}{3}$$

$$\sim \frac{1}{3n^2+n}$$

$$\frac{\overset{(2n+1)\,2n}{\cancel{(2n+1)!}}}{\cancel{(2n-1)!}\,\underbrace{(2n+1-(2n-1))!}_{2!}} = \frac{4n^2+2n}{2} = 2n^2+n \sim 2n^2$$

2)

$$\lim_{n\to+\infty} \frac{e^{\frac{3n^2+n}{2n^3+1}} - 1 - \operatorname{sen}\frac{3n+1}{2n^2-1}}{n^\alpha} =$$

$$\sim \frac{3n^2+n}{2n^3+1} \qquad\qquad \sim \frac{3n+1}{2n^2+1}$$

$$\text{Num} \sim \frac{3n^2+n}{2n^3+1} - \frac{3n+1}{2n^2-1} =$$

$$= \frac{(3n^2+n)(2n^2-1)-(3n+1)(2n^3+1)}{(2n^3+1)(2n^2-1)}$$

$$= \frac{\cancel{6n^4} - n^3 + \dots - \cancel{6n^4} - 2n^3 + \dots}{4n^5+\dots} \sim -\frac{3n^3}{4n^5}$$

$$\Rightarrow \sim -\frac{3}{4}\frac{1}{n^{\alpha+2}} \quad \begin{cases} \alpha=-2 \Rightarrow -3/4 \\ \alpha>-2 \Rightarrow 0 \\ \alpha<-2 \Rightarrow -\infty \end{cases}$$

3) $\sum_{n=2}^{+\infty} \left(\frac{\pi}{e^2}\right)^n$ converge

basti osservare che $\frac{\pi}{e^2} < 1$

quindi studiarla come geometrica con ragione <1

Ma $\sum_{0}^{\infty} r^n = \frac{1}{1-r}$

allora $1 + \frac{\pi}{e^2} + \underline{\left(\frac{\pi}{e^2}\right)^2 + \dots} = \frac{1}{1-\frac{\pi}{e^2}}$

quindi la somma è $\frac{1}{1-\frac{\pi}{e^2}} - 1 - \frac{\pi}{e^2}$

4) $\sum \frac{n^\alpha \log n}{n^2 \log^3 n + 1}$

ha il termine generale confrontabile con

$$\frac{1}{n^{2-\alpha} \log^2 n}$$

$\Rightarrow$ considerando l'armonica generalizzata

che converge per $2-\alpha > 1$

diverge per $2-\alpha < 1$

per $2-\alpha = 1$ poiché ho $\log^2 n$ converge