MdC - 24/1/2014 2° comp.

1) $\lim_{n \to +\infty} n \cdot \left( e^{\frac{2}{3n}} - 1 \right)$

2) $\lim_{n \to +\infty} \left( \frac{n+2}{n+3} \right)^n$

3) $\lim_{n \to +\infty} n \cdot \left( \sqrt{1 - \frac{3}{2n}} - \sqrt[3]{1 + \frac{2}{3n}} \right)$

4) $\lim_{n \to +\infty} \frac{\left( \operatorname{tg} \frac{2}{n} + 1 \right)^2 - 1}{\frac{4}{3n} + \frac{1}{n^2+1}}$

5) $\lim_{n \to +\infty} \frac{\log \left( 1 - \frac{3n - \operatorname{sen} n}{n^2+1} \right)}{e^{\frac{2n}{n^2+1}} - 1}$

6) $\sum \left( \frac{1}{\sqrt{n} - 4} - \frac{1}{\sqrt{n} + 4} \right)$

7) $\sum \frac{(n!)^2}{n^3 (2n)!}$

2° comp 24/1/2014

1) $e^{\frac{2}{3n}} - 1 \sim \frac{2}{3n}$

2) $\left(1 + \frac{-1}{n+3}\right)^{-(n+3)} \cdot \frac{-n}{n+3} \sim e^{-1}$

$\left(1 + \frac{-1}{n+3}\right)^{-(n+3)} \to e$, $\frac{-n}{n+3} \to -1$

3) $\infty \cdot 0 \qquad n\left(\underbrace{\left(1-\frac{3}{n}\right)^{1/2} - 1}_{\sim -\frac{1}{2}\cdot\frac{3}{n}} - \left(\underbrace{\left(1+\frac{2}{3n}\right)^{1/3} - 1}_{\frac{1}{3}\cdot\frac{2}{n}}\right)\right)$

4) $\left(\operatorname{tg}\frac{2}{n} + 1\right)^2 - 1 \sim \left(\frac{2}{n} + 1\right)^2 - 1 =$

$= \frac{4}{n^2} + \frac{4}{n} + 1 - 1 \sim \frac{4}{n}$

$\frac{4}{3n} + \frac{1}{n^2+1} \sim \frac{4}{3n}$

5) $\log\left(1 - \frac{3n - \operatorname{sen} n}{n^2+1}\right) \sim \log\left(1 - \frac{3}{n}\right) \sim -\frac{3}{n}$

$e^{\frac{2n}{n^2+1}} - 1 \sim e^{\frac{2}{n}} - 1 \sim \frac{2}{n}$

6) $a_n = \frac{\sqrt{n+4} - \sqrt{n+4}}{(\sqrt{n}-4)(\sqrt{n}+4)} = \frac{8}{n-16} \sim \frac{8}{n}$

7) $\frac{(n+1)^2 (n+1)!}{(n+1)^3 (2n+2)!} \cdot \frac{n^3 (2n)!}{(n!)^2} = \frac{n^3}{(n+1)(2n+2)(2n+1)} \to \frac{1}{4}$

$(2n+2)(2n+1)$