MdC ④ 23/5/14

Cognome Nome Matr

1) Scrivere la funzione generatrice associata a

$$a_n = 3 \cdot 5^{n-1} - 4 \cdot 3^{n+1}$$

2) $\{a_n\} = \left\{ \frac{1}{2n+1} \operatorname{sen} \frac{n^2 x}{3n+1} \right\}$

converge? e se si converge uniformemente?

3) Determinare il coeff di x^5 nello sviluppo di Mc Laurin di

$$y = \operatorname{sen}(3x^2 - x) - \operatorname{Sh}(2x)$$

4) $\begin{cases} x y' - (x+1) y = 0 \\ y(1) = 2 \end{cases}$

5) $\displaystyle\int_{\pi/6}^{\pi/3} \frac{x}{\cos^2 x}\, dx$ 6) $\displaystyle\int \frac{x^3+1}{(x+1)(x^2-4)}\, dx$

7) $\displaystyle\int_0^{-\infty} (x^2 - x) e^{2x}\, dx$ 8) $\displaystyle\int_e^{+\infty} \frac{1}{x \log(2x)}\, dx$

④ 23/5/14

1) $a_n = 3\cdot 5^{n-1} - 4\cdot 3^{n+1}$

$$\sum_0^\infty a_n x^n = \sum_0^\infty \left(3\cdot 5^{n-1} - 4\cdot 3^{n+1}\right) x^n =$$

$$= (3/5)\cdot \sum_0^\infty 5^n x^n - 4\cdot 3\cdot \sum_0^\infty 3^n\cdot x^n =$$

$$= \frac{3}{5}\cdot \frac{1}{1-5x} - 12\cdot \frac{1}{1-3x}$$

2) $\{a_n\} = \left\{\frac{1}{2n+1} \operatorname{sen} \frac{n^2 x}{3n+1}\right\}$

$$\left|\operatorname{sen} \frac{n^2 x}{3n+1}\right| \leq 1$$

$$a_n \leq \frac{1}{2n+1} \to 0 \text{ per } n \to +\infty$$

$\Rightarrow$ converge a $y=0$

inoltre comunque scelto $\bar{x}$ per $n \geq \bar{n}$ opportuno $a_n(\bar{x})$ è arbitrariamente piccola $\Rightarrow$ conv. unif.

3) $y = \operatorname{sen}(3x^2 - x) - \operatorname{Sh}(2x)$

coeff di x^5

$$y = (3x^2 - x) - \frac{(3x^2-x)^3}{3!} + \frac{(3x^2-x)^5}{5!} + \ldots$$

$$- \left(2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots\right)$$

$$-\frac{3(3x^2)^2\cdot(-x)}{3!} \qquad \frac{-x^5}{120} \qquad -\frac{32x^5}{120}$$

$$\frac{27}{6}x^5$$

$$\frac{27}{6} - \frac{1}{120} - \frac{32}{120}$$

4) $\begin{cases} xy' - (x+1)y = 0 \\ y(1) = 2 \end{cases}$

Variabili separabili $\dfrac{dy}{y} = \dfrac{x+1}{x}\,dx$

$\log|y| = x + \log|x| + c$

$\log 2 = 1 + \log 1 + c \Rightarrow c = \log 2 - 1$

$\log|y| = x + \log|x| + \log 2 - 1$

5) $\displaystyle\int_{\pi/6}^{\pi/3} \frac{x}{\cos^2 x}\,dx \qquad \int \frac{x}{\cos^2 x}\,dx = \int x \cdot d\,\operatorname{tg} x = \cdots$

6) $\displaystyle\int \frac{\cancel{x^3+1}^{\,x^2-x+1}}{\cancel{(x+1)}(x^2-4)}\,dx = \int \frac{x^2-x+1}{x^2-4}\,dx =$

$$\begin{array}{r|l} x^2 - x + 1 & x^2-4 \\ \cline{2-2} -x^2 + 4 & 1 \\ \hline -x + 5 & \end{array}$$

$= \displaystyle\int \left(1 + \frac{-x+5}{x^2-4}\right) dx = \ldots$

$\dfrac{A}{x-2} + \dfrac{B}{x+2} = \dfrac{-x+5}{x^2-4}$

7) $\displaystyle\int_0^{-\infty} (x^2-x)\,e^{2x}\,dx = \int_0^{-\infty} \frac{1}{2}(x^2-x)\,d\,e^{2x} =$

$= \left[\frac{1}{2} e^{2x}(x^2-x)\right]_0^{-\infty} - \displaystyle\int_0^{-\infty} \frac{1}{2} e^{2x}(2x-1)\,dx =$

poi ancora per parti

8) $\displaystyle\int_e^{+\infty} \frac{1}{x\log(2x)}\,dx$ porre $\log(2x) = t$