

Soluzioni ③ compitino online

Soluzioni ① compitino online

19/11/2016

3. ①  $2x > \sqrt{x^2 - 7x + 6}$

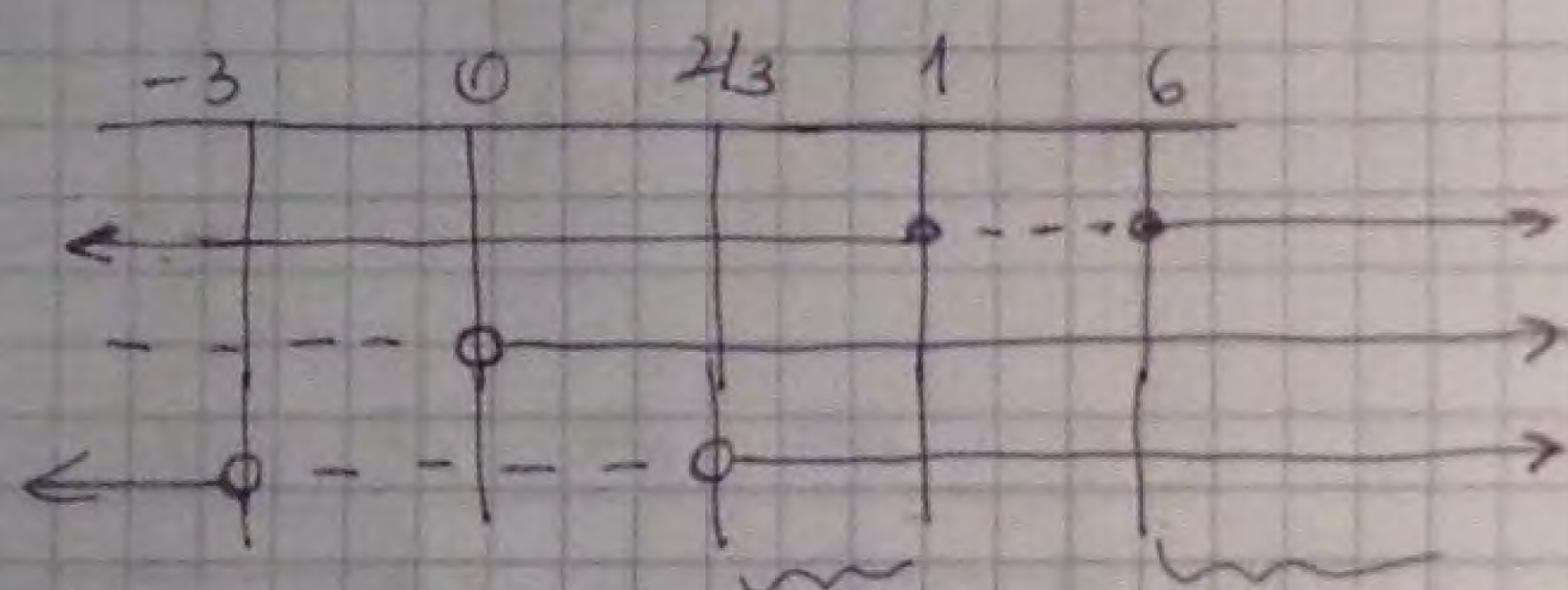
$$x^2 - 7x + 6 \geq 0$$

$$(x-6)(x-1) \geq 0$$

$$x \leq 1 \vee x \geq 6 \quad \text{C.E.} \quad \textcircled{1}$$

$$\begin{cases} f(x) \geq 0 \\ g(x) > 0 \rightarrow 2x > 0 \rightarrow x > 0 \\ f(x) < g^2(x) \rightarrow x^2 - 7x + 6 < 4x^2 \\ 3x^2 + 7x - 6 > 0 \end{cases} \quad \textcircled{0.5}$$

$$x_{1/2} = \frac{-7 \pm \sqrt{49 - 4 \cdot 3 \cdot (-6)}}{6} = \frac{-7 \pm \sqrt{121}}{6} = \frac{-7 \pm 11}{6} = \begin{cases} 2/3 \\ -3 \end{cases} \quad \textcircled{0.5}$$



$$\frac{2}{3} < x \leq 1 \quad x \geq 6$$

$$\text{sol: } \left(\frac{2}{3}, 1\right] \cup [6, \infty) \quad \textcircled{1}$$

4. ②  $\frac{\log(2x+1) + 2}{4e^{2x} - 9} \leq 0$

$$2x+1 > 0 \quad \boxed{x > -1/2} \quad \text{C.E.} \quad \textcircled{0.5}$$

$$4e^{2x} - 9 \neq 0$$

$$e^{2x} \neq 9/4$$

$$2x \neq \ln(9/4)$$

$$2x \neq 2\ln(3/2)$$

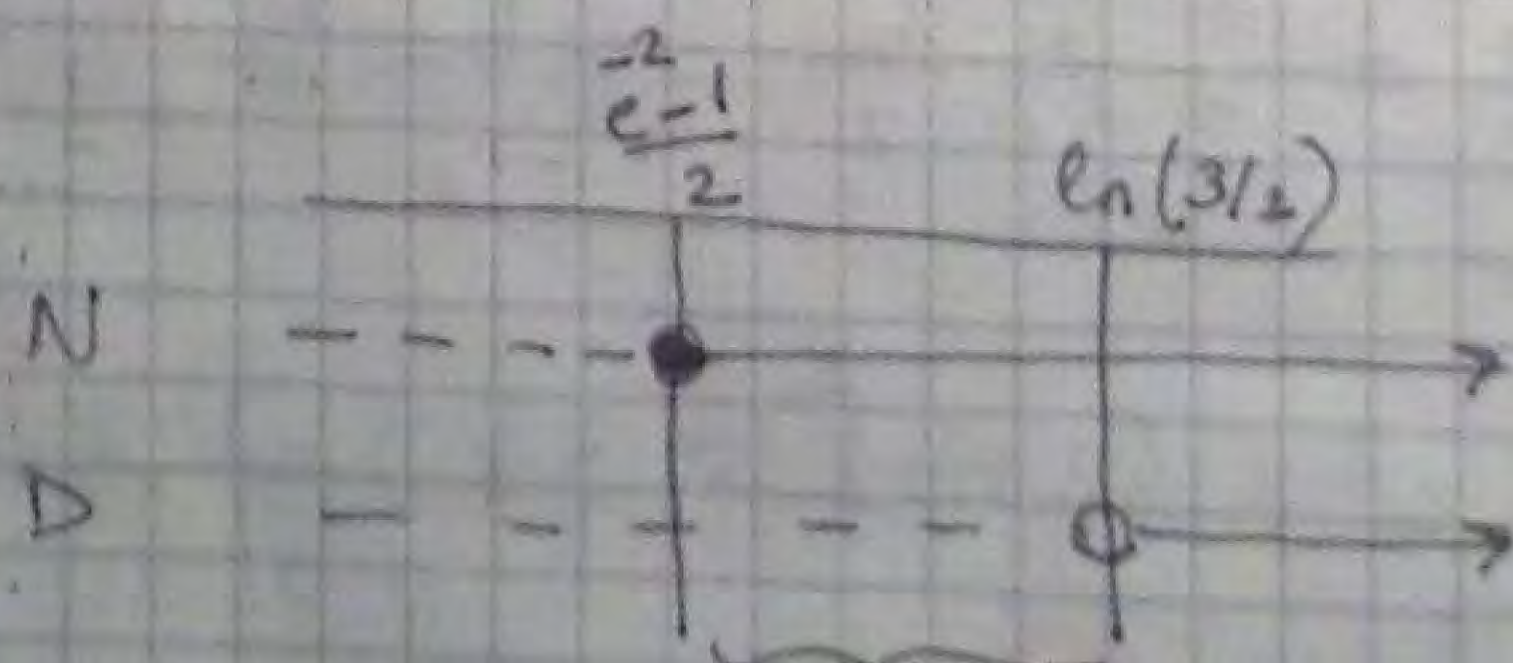
$$x \neq \ln(3/2) \quad \text{C.E.} \quad \textcircled{0.5}$$

$$N: \log(2x+1) + 2 \geq 0$$

$$\log(2x+1) \geq -2$$

$$2x+1 \geq e^{-2} \quad \textcircled{1}$$

$$\boxed{x \geq \frac{e^{-2} - 1}{2}} = \frac{-2}{2} - \frac{1}{2}$$



$$\frac{e^{-2} - 1}{2} \leq x < \ln(3/2)$$

$$\text{C.E. } 0 \quad \frac{e^{-2} - 1}{2} \quad \ln(3/2)$$

soluzione ①

$$D: 4e^{2x} - 9 > 0 \Rightarrow e^{2x} > \frac{9}{4}$$

$$e^{2x} - \frac{9}{4} > 0$$

$$(e^x + \frac{3}{2})(e^x - \frac{3}{2}) = 0$$

$$e^x < -\frac{3}{2} \vee e^x > \frac{3}{2}$$

$$\text{No! } x > \ln(3/2)$$

$$\ln e^{2x} > \ln(\frac{9}{4})$$

$$2x > \ln(\frac{9}{4})$$

$$2x > 2\ln(3/2)$$

$$x > \ln(3/2)$$

①



$$4 \textcircled{3} \quad 2 \log_e(x) - 3 < \frac{2 \log_e(x) + 3}{\log_e(x)}$$

$$\log_e = \ln$$

$$2 \ln(x) - 3 - \left( \frac{2 \ln(x) + 3}{\ln(x)} \right) < 0$$

$$\frac{2 \ln^2(x) - 5 \ln(x) - 3}{\ln(x)} < 0$$

$$t = \ln x$$

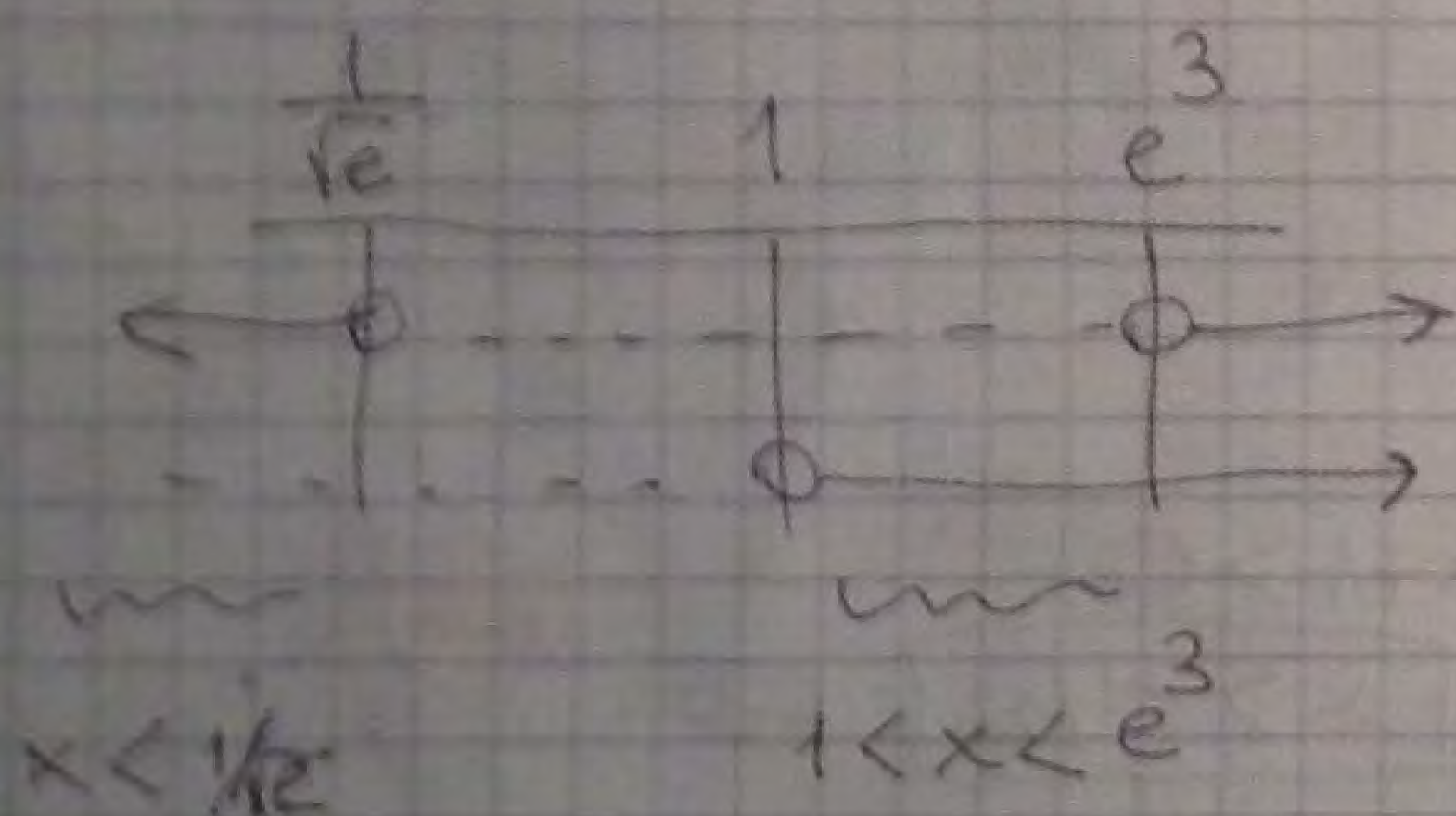
$$N) \quad 2t^2 - 5t - 3 > 0$$

$$(2t+1)(t-3) > 0$$

$$t = -1/2 \quad t = 3$$

$$\ln x < -1/2 \quad \ln x > 3$$

$$x < e^{-1/2} \quad x > e^3$$



$$D) \quad \ln x > 0$$

$$x > 1$$

$$\left( \begin{array}{l} e > 1 \\ \sqrt{e} > \sqrt{1} \\ \frac{1}{\sqrt{e}} < 1 \end{array} \right)$$

$$C.E. \quad x > 0$$

$$\text{Sol} : 0 < x < 1/\sqrt{e} \vee 1 < x < e^3$$

$$x \in \left( 0, \frac{1}{\sqrt{e}} \right) \cup \left( 1, e^3 \right)$$

$$3 \textcircled{5} \quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} = \frac{a}{2} \Rightarrow \boxed{a = \sqrt{3}}$$

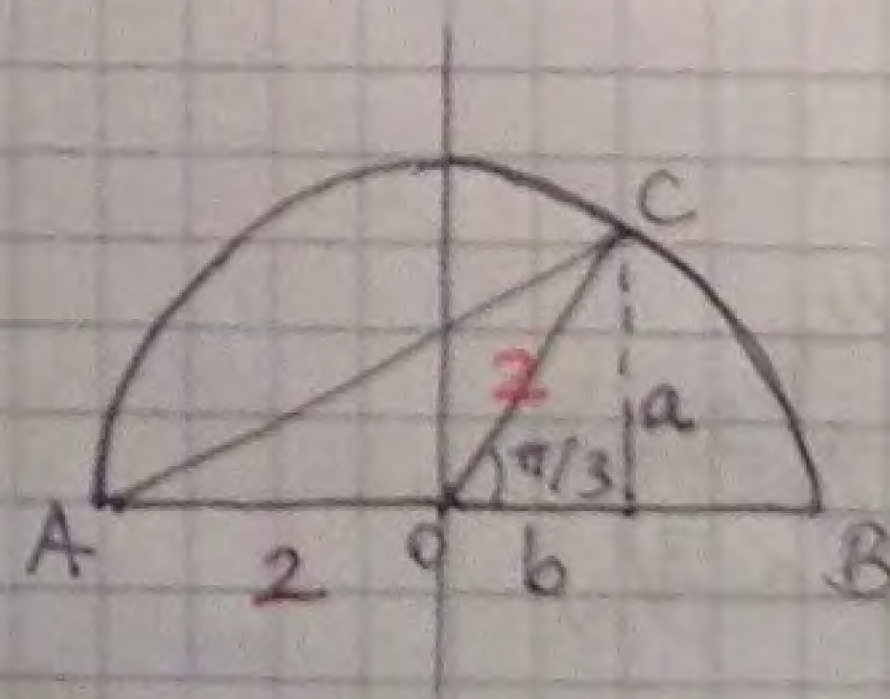
$$\text{Area} = \frac{2 \cdot \sqrt{3}}{2} = \sqrt{3}$$

$$\cos \frac{\pi}{3} = \frac{1}{2} = \frac{b}{2} \Rightarrow b = 1$$

$$AC^2 = a^2 + (2+b)^2 = 3 + 9 = 12$$

$$AC = 2\sqrt{3}$$

$$P = 2\sqrt{3} + 2 + 2 = 2\sqrt{3} + 4$$



$$\sin \frac{\pi}{3} = \frac{a}{2}$$

$$\cos \frac{\pi}{3} = \frac{b}{2}$$



4

$$\underbrace{\cos(x)}_{1 \text{ fact.}} \underbrace{[\tan(x) - 1]}_{2 \text{ fact.}} > 0$$

$$\text{C.E.: } x \neq \frac{\pi}{2}, x \neq \frac{3\pi}{2}$$

0.5

1)  $\cos(x) > 0$

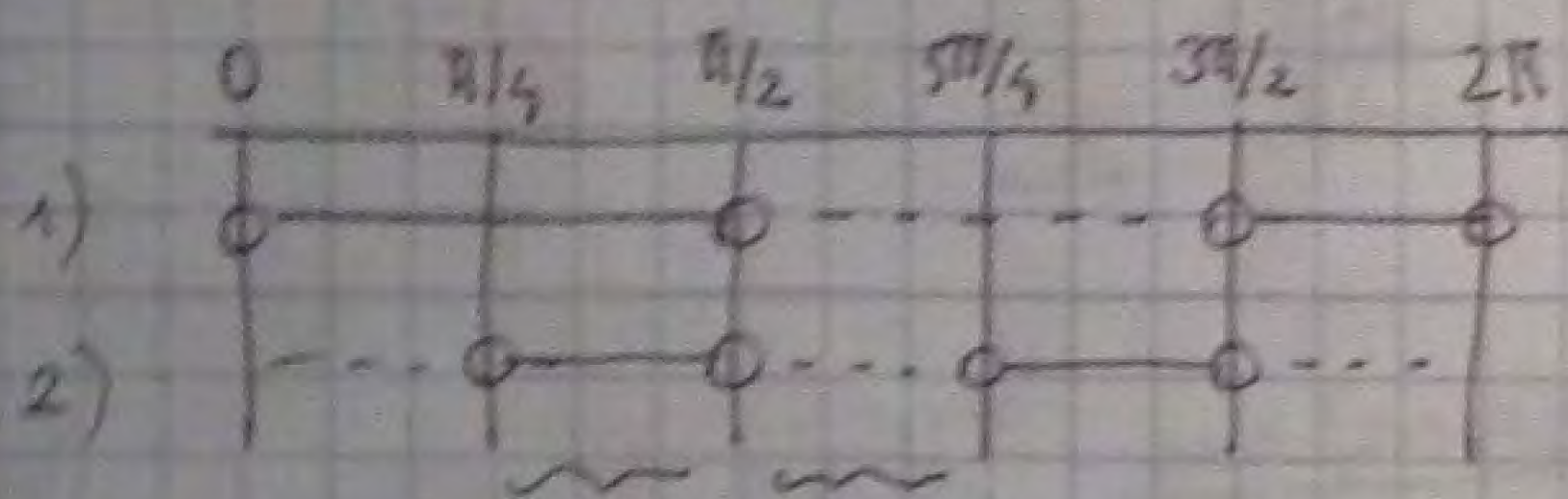
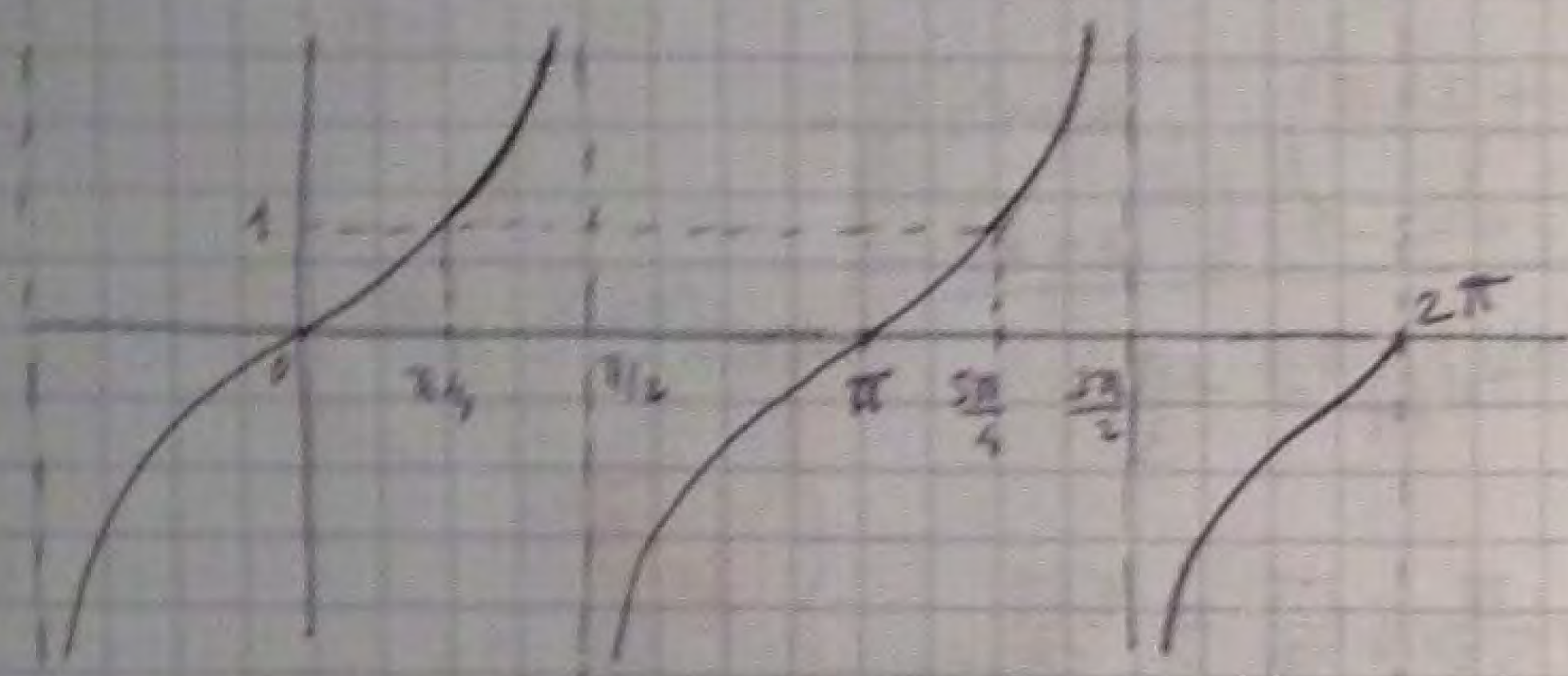
$$0 < x < \frac{\pi}{2} \vee \frac{3\pi}{2} < x < 2\pi$$

1

2)  $\tan(x) - 1 > 0 \Rightarrow \tan(x) > 1$

$$\frac{\pi}{4} < x < \frac{\pi}{2} \vee \frac{5\pi}{4} < x < \frac{3\pi}{2}$$

1



$$\frac{\pi}{4} < x < \frac{\pi}{2} \vee \frac{5\pi}{4} < x < \frac{3\pi}{2}$$

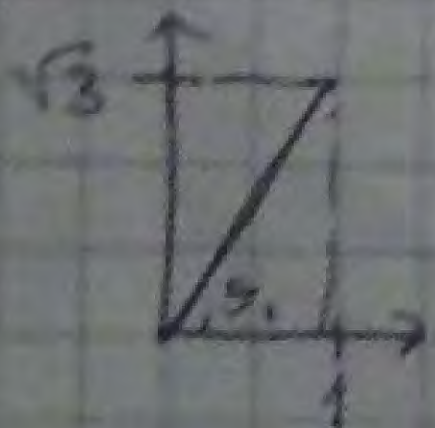
$$\Rightarrow \text{sol: } \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$$

1.5

5 (6)  $z = \frac{z_1^{21}}{z_2^{18}} = \frac{(1 + \sqrt{3}i)^{21}}{(\sqrt{3} + i)^{18}}$

$$z_1 = 1 + \sqrt{3}i$$

$$r_1 = \sqrt{1+3} = 2$$



$$\tan \theta_1 = \sqrt{3}$$

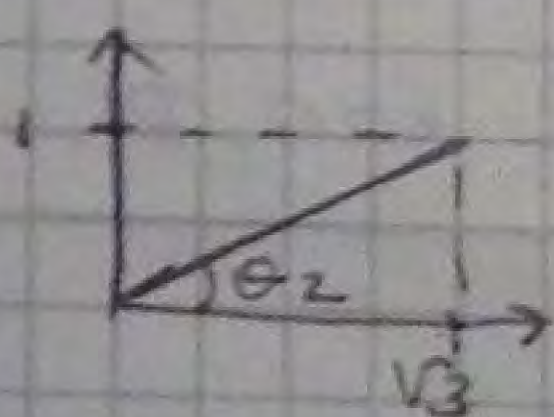
$$\theta_1 = \arctan(\sqrt{3})$$

$$\arg z_1 = \theta_1 = \pi/3$$

$$z_1 = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$z_2 = \sqrt{3} + i$$

$$r_2 = \sqrt{3+1} = 2$$



$$\tan \theta_2 = \frac{1}{\sqrt{3}}$$

$$\theta_2 = \arctan\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \theta_2 = \pi/6$$

$$\arg z_2 = \pi/6$$

$$z_2 = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

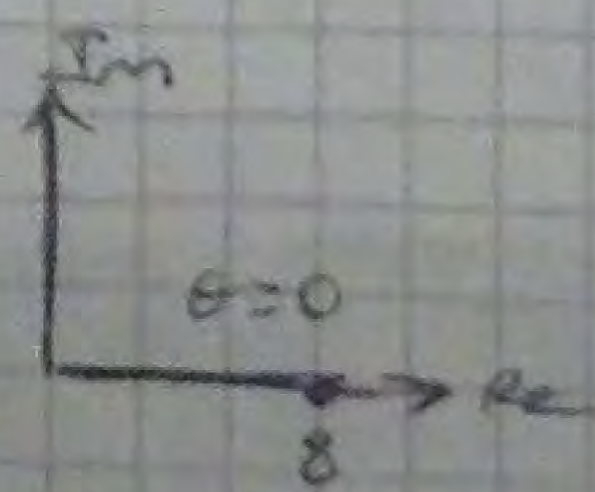
$$z_1^{21} = 2^{21} \left( \cos \frac{\pi}{3} \cdot 21 + i \sin \frac{\pi}{3} \cdot 21 \right) = -2^{21}$$

$$z_2^{18} = 2^{18} \left( \cos \frac{\pi}{6} \cdot 18 + i \sin \frac{\pi}{6} \cdot 18 \right) = -2^{18}$$

$$z = \frac{-2^{21}}{-2^{18}} = 2^3 = 8$$

0.5

$$z = 8 \Rightarrow |z| = 8, \arg z = 0 \Rightarrow z = 8(\cos 0 + i \sin 0)$$

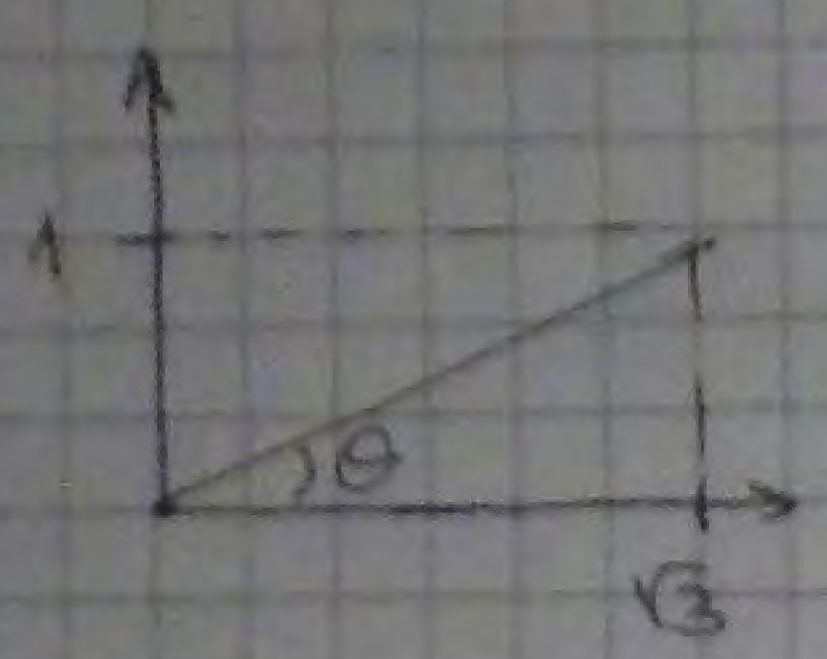




5+2

(7)  $z = \sqrt{3} + i$

$|z| = \sqrt{3+1} = 2$



$\arg z = \theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

0.5

(1.5)  $\sqrt[3]{z} = \sqrt[3]{2} \left( \cos \frac{\pi/6 + 2k\pi}{3} + i \sin \frac{\pi/6 + 2k\pi}{3} \right), k=0,1,2$

$z_0 = \sqrt[3]{2} \left( \cos \frac{\pi}{18} + i \sin \frac{\pi}{18} \right)$

Diff =  $\frac{2\pi}{3}$

$z_1 = \sqrt[3]{2} \left( \cos \frac{13\pi}{18} + i \sin \frac{13\pi}{18} \right)$

$\frac{\pi}{18} + \frac{2\pi}{3} = \frac{13\pi}{18}$

$z_2 = \sqrt[3]{2} \left( \cos \frac{25\pi}{18} + i \sin \frac{25\pi}{18} \right)$

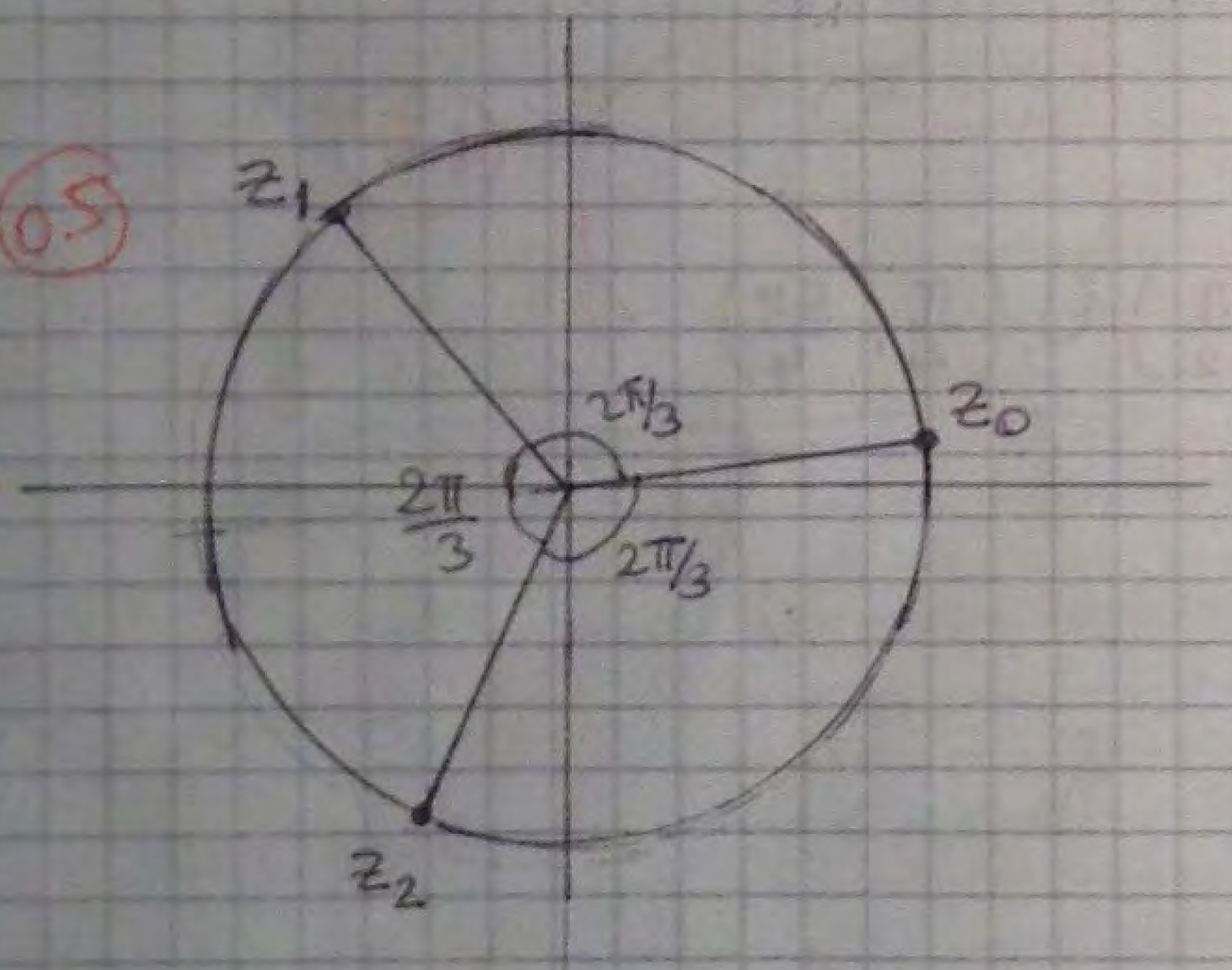
$\frac{13\pi}{18} + \frac{2\pi}{3} = \frac{25\pi}{18}$

2.5

same arg for

1.5

0.5

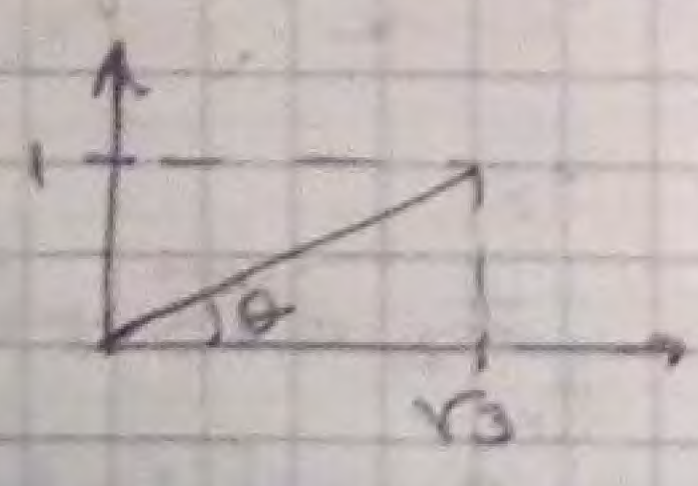


$z = \sqrt{3} + i$

0.5

$\ln(\sqrt{3} + i) = \ln|\sqrt{3} + i| + i[\arg(\sqrt{3} + i) + 2k\pi], k \in \mathbb{Z}$

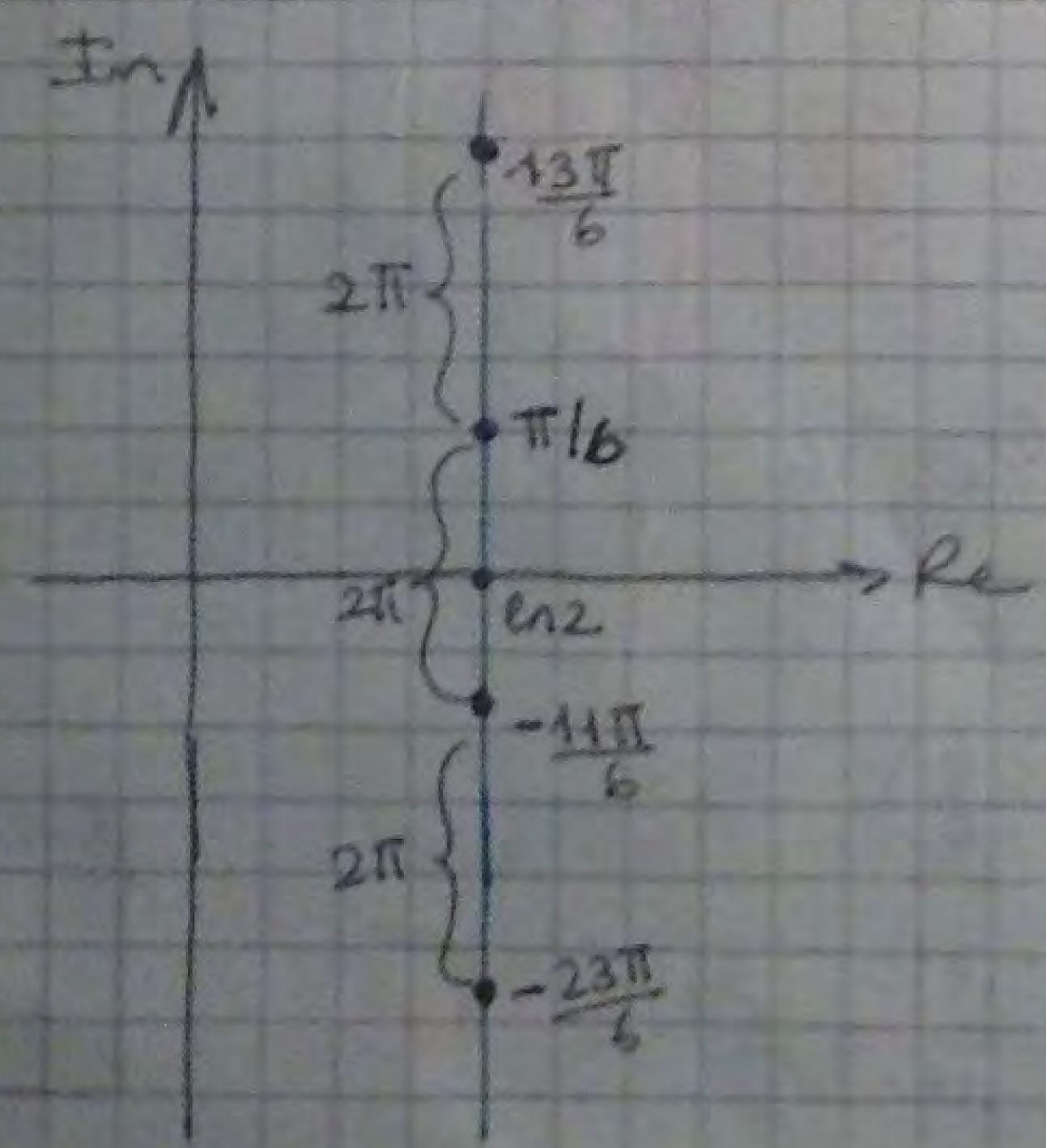
$|\sqrt{3} + i| = \sqrt{3+1} = 2$



$\arg(\sqrt{3} + i) = \pi/6$

(1)  $\ln(\sqrt{3} + i) = \ln 2 + i\left(\frac{\pi}{6} + 2k\pi\right), k=0, \pm 1, \dots$

0.5



$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$

$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

$-\frac{11\pi}{6} - 2\pi = -\frac{23\pi}{6}$